Alcubierre’s Warp Drive: Problems and Prospects

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Abstract. Alcubierre’s warp drive geometry seemingly represents the ultimate dream for interstellar travel: there is no speed limit, the passengers are weightless whatever the acceleration, and there is no time dilation. However, in its original form, the proposal suffers from several fatal flaws, such as unreasonably high energies, energy moving in a locally spacelike direction, and a violation of the energy conditions of classical Einstein gravity. I present a possible solution for one of these problems, and I suggest ways to at least soften the others.

INTRODUCTION

Alcubierre introduced an entirely novel concept for superluminal travel (Alcubierre, 1994). Put simply, the idea was to start from Minkowski spacetime, choose an almost arbitrary curve, and deform spacetime in the immediate vicinity in such a way that the curve became a timelike geodesic. This implied that anyone traveling on such a curve would be weightless, whatever the acceleration with respect to observers in the flat space outside. The metric is

\[ ds^2 = -dt^2 + (dx - v(t)f(r(t, x, y, z))dt)^2 + dy^2 + dz^2, \]

where \( r = \sqrt{(x - x_c(t))^2 + y^2 + z^2} \), with \( x_c(t) \) the path of the central geodesic; \( v(t) = \frac{dx_c(t)}{dt} \) is the apparent velocity of the warp drive. The function \( f(r) \) is zero outside the warp bubble and 1 inside. The passengers of a warp drive would live in a bubble of flat spacetime, surrounded by a ‘wall’ in which space is distorted in such a way that it contracts in the front and expands in the back, thus at the same time ‘pushing’ and ‘pulling’ the bubble. The proper time of the central geodesic would be the same as the external coordinate time, so that there would be no time dilation. And the most interesting feature was, of course, the apparent absence of any speed limit.

This sounded promising for interstellar travel, but the warp drive has its share of troubles. I will try to provide a comprehensive outline of the problems that have been identified over the past five years, and look at possible solutions. I will keep the discussion on a descriptive level, and not overload it with formulas. The detailed calculations can be found in the references.

PROBLEMS

Alcubierre himself immediately pointed out that his geometry suffered from a problem that also plagues traversable wormholes: a violation of at least three of the energy conditions of general relativity, specifically the weak, strong, and dominant energy conditions (WEC, SEC, and DEC). These conditions essentially impose positivity on the energy density/pressures; for instance, the WEC demands that the energy density measured locally by any observer should be positive at any time. See (Hawking, 1987) for an overview. Energy that violates the energy conditions is usually called ‘exotic matter’.
Although the energy conditions have been used as input for many important theorems of classical general relativity in the past (Hawking, 1987) provides a host of examples), their physical validity is no longer credible. For example, the WEC, SEC and DEC are violated by the well known Casimir effect (Casimir, 1948), although the violations are tiny. In its simplest realization, the Casimir effect entails a difference between the zero point energy of the electromagnetic vacuum energy in between and outside two parallel conducting plates. The boundary conditions imposed by the plates restrict the number of zero modes in between, resulting in an energy density that is lower than that of the vacuum in the absence of the plates. One consequence of the effect, an attractive force between the plates, was first shown experimentally more than forty years ago (Spaarnaay, 1958). Other quantum effects are known to violate energy conditions (Visser, 1995).

Even though the energy conditions are thought to be violated in Nature, there might be restrictions on the extent of the violations. Ford and Roman (Ford, 1996) have argued that WEC violations originating from vacuum fluctuations of quantum fields are subject to an uncertainty principle: if $\rho$ is the energy density measured by an inertial observer, averaged over a 'sampling time' $\tau_0$, where $\tau_0$ is much smaller than the local curvature radius of spacetime divided by $c$, then

$$\bar{\rho}\tau_0^4 \geq -\frac{3\hbar}{32\pi^2 c^3}$$

(2)

Considering that $\frac{\hbar}{c^3} \approx 3 \times 10^{-60} \text{J} \cdot \text{m}^4$, and taking into account that the fourth power of the sampling time appears, this is a severe restriction.

Using this so-called quantum inequality (QI), Ford and Pfenning (Ford, 1997) showed that the warp bubble wall should be no thicker than a hundred Planck lengths for velocities in the order of $c$; otherwise, inertial observers crossing the wall would measure negative energy for too long. In addition, they calculated that the total energy of a macroscopic warp bubble is given by

$$E = -\frac{1}{12}v^2\frac{c^2 R^2}{G\Delta},$$

(3)

where $R$ is the bubble radius, $\Delta$ its wall thickness, and $v$ the apparent velocity of the warp drive. For a bubble with a radius of 100 m, a wall thickness of 100 $l_P$ and a velocity $v \sim c$, one gets roughly

$$E \sim -10^{63} \text{kg} \times c^2,$$

(4)

which, in absolute value, is ten orders of magnitude larger than the energy content of the entire visible Universe.

Some physicists might raise yet another objection, namely that the basic warp drive geometry can easily be used for creating a spacetime with closed timelike curves (CTCs) (Everett, 1996). Indeed, it is easy to see that any means of superluminal travel within an asymptotically flat spacetime will appear to be time travel for some observers in the quasi-Minkowskian region: when an interval is crossed which is considered spacelike by a quasi-Minkowskian observer, there will always be an observer for whom the arrival takes place before the departure. Hawking's Chronology Protection Conjecture (Hawking, 1991) forbids CTCs, but it has never been derived from first principles; rather, it is based on a number of examples suggesting that quantum fluctuations in the energy density of fields will diverge in the presence of CTCs. An explicit counterexample – at least for the case of non-gravitational fields – is provided in (Li-Xin Li, 1994). Of course, the absence of divergences in itself does not guarantee that CTCs are allowed by Nature. The debate continues.

However, the original Alcubierre geometry has a much more fundamental problem. This is the behaviour of negative energy density in the outer layers of the warp bubble. It is easy to see that, outside some surface in the warp bubble wall (the 'critical surface'), the energy will move with spacelike velocity, i.e. it will move locally faster than light (Krasnikov, 1998; Coule, 1998). This is forbidden both by classical general relativity and quantum field theory. For reasons that will become clear later on, it has (rather euphemistically) been
referred to as the ‘control problem’ (Everett, 1997; Krasnikov, 1998); here, I will call it the ‘tachyonic motion problem’ (TMP). It would seem that solving the TMP is the make-or-break issue for ‘warp drive engineering’.

**TENTATIVE SOLUTIONS**

By now it should be clear that Alcubierre’s proposal, when taken literally, is not realistic. However, Alcubierre himself considered his geometry to be no more than an ansatz. I will try to show that altering the spacetime in a judicious way can bring us a long way in solving the problems.

First, let us have a look at the total energy problem. In (Van Den Broeck, 1999a), it was shown that altering the Alcubierre geometry in a simple way could dramatically reduce the energy required. Basically, the idea was to keep the warp bubble subatomically small, so that its energy would be small, but ‘blow up’ its interior, so that a large spaceship would fit inside. The modified geometry is defined by

\[ ds^2 = -dt^2 + B^2(r)\left[(dx - v(t)f(r(t, x, y, z))dt)^2 + dy^2 + dz^2\right], \]  

(5)

where the function \( B \) determines the space blow-up. \( B \) is chosen such that it is identically 1 outside the warp bubble and in the wall, becoming very large towards the center, and then leveling off to allow for a flat interior. The general structure of such a modified warp drive is shown in Figure 1.

Calculating the energy density as measured by inertial observers and integrating over a constant time hypersurface, one finds that the blow-up requires an amount of positive energy of the order of a solar mass, and an amount of negative energy of the same magnitude. This puts the modified Alcubierre geometry in the energy bracket of a large traversable wormhole (Visser, 1995).

An important problem is that the energy densities (in absolute value) of both the bubble wall and the space blow-up region, attain values of no less than \( 10^{24} \) kg/m\(^3\). In (Van Den Broeck, 1999a), great care was taken that the QI would not be violated. Even so, a mechanism for creating such large densities is sorely lacking; all the effects we know of that violate energy conditions are microscopic quantum effects in density as well as in size. For the blow-up, it might still be possible to lower the density by making a better choice for \( B \) than (??), but this is not the case for the bubble wall, since its energy density in e.g. a plane through

![FIGURE 1](image)

**FIGURE 1.** Region I is the ‘pocket’, which has a large inner metric diameter. II is the transition region from the blown-up part of space to the ‘normal’ part. It is the region where \( B \) varies. From region III outward we have the original Alcubierre metric. Region IV is the wall of the warp bubble; this is the region where \( f \) varies. Spacetime is flat, except in the shaded regions.
FIGURE 2. A schematic representation of a warp bubble. The function $f$ varies in the shaded region. The dotted circle represents the critical surface, while the thick line is the horizon. The region where $H$ varies is not drawn.

FIGURE 3. An independently moving warp bubble. The dotted line again represents the critical sphere; the thick line now indicates a singular surface.
the center and perpendicular to the direction of motion is

\[ T_{00} = \frac{v^2}{32\pi} \left( \frac{df(r)}{dr} \right)^2, \tag{6} \]

and because of the fact that the wall is constrained to be so thin, \( df/dr \) must become large somewhere.

In the previous section, I referred to the TMP as the most important problem of warp drive geometries. There exists a 'critical surface' such that all negative matter outside the sphere moves tachyonically. A solution was hinted at in (Van Den Broeck, 1999a). There, it was noted that superluminal motion can be simulated. A pulsar provides a simple example: the spot where the pulsar beam intersects a sphere with a sufficiently large radius will appear to move faster than light. In the warp drive, a signal could be sent to a point in the bubble wall, diminishing the exotic matter density, and another signal to a point further ahead, spacelike separated from the first, where the density should be increased. Now, some parts of the warp bubble wall are inaccessible for signals sent by the 'pilot' of the warp bubble, due to a horizon that cuts him off from the front of the bubble (see Figure 2).

One might draw the conclusion that the warp drive will need continuous support from outside, possibly by devices placed in advance along the path of the bubble. However, this is not necessary; if the pilot keeps controlling the parts of the bubble that are accessible to him, the Alcubierre geometry will certainly be maintained behind the horizon. The exotic matter in front of the horizon will, in a short time, be overtaken by the superluminally moving bubble. This will lead to a surface singularity; at the horizon, there will be a discontinuous transition from Minkowski spacetime (in front) to Alcubierre spacetime (behind). The result is shown schematically in Figure 3.

The surface singularity will not inconvenience the pilot; once the flight is over, he can 'engineer' the back of the warp bubble in such a way that he drops out of it, never encountering the singularity (which he wouldn’t be able to reach, even if he wanted to). It should be emphasized that at present, there is no practical proposal for implementing the control process described above. This is work for the future.

**CONCLUSIONS**

Apparently, the theoretical problems with the warp drive do not seem to be insurmountable. Now, how soon will we be able to build one? I've discussed the TMP in some detail above, so here I’ll focus on the other problems.

To avoid unphysically large energies, the warp bubble has to be kept subatomically small, so we would have to learn how to manipulate spacetime on such small scales. Let me add immediately that the energy problem is a consequence of the fact that the bubble wall needs to be thin, which is imposed by the QI. However, the validity of the QI in highly curved spacetimes is not beyond dispute; after all, it was not proven rigorously. Should the QI be false, then it is clear from expression (3) that the absolute value of the total energy can be made as small as one pleases, and the construction of (Van Den Broeck, 1999a) would be unnecessary. However, if they are valid, it seems we would need to have access to large amounts of positive as well as negative energies.

To generate the negative energy densities, we will need some quantum effect, and it will need to be a macroscopic one. It is not clear how e.g. the Casimir effect can be made large. The quantum vacuum has not yet yielded to our desires, but there is no reason why that couldn’t happen in the foreseeable future.

What all this boils down to is: chances are slim that the warp drive will be built in our lifetime. But science has a whole new millennium ahead of it...
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REFERENCES