# Supersymmetry Breaking Casimir Warp Drive

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**Abstract.** This paper utilizes a recent model which relates the cosmological constant to the Casimir energy of the extra dimensions in brane-world theories. The objective of this paper is to demonstrate that, given some sufficiently advanced civilization with the ability to manipulate the radius of the extra dimension, a local adjustment of the cosmological constant could be created. This adjustment would facilitate an expansion/contraction of the spacetime around a spacecraft creating an exotic form of field-propulsion. This idea is analogous to the Alcubierre bubble, but differs entirely in the approach, utilizing the physics of higher dimensional quantum field theory, instead of general relativity.

**Keywords:** Warp drive, Casimir effect, supersymmetry, brane-world, Alcubierre.
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## INTRODUCTION

Over the last decade there has been theoretical interest in curiosities dubbed 'warp-drives' initiated by the 1994 paper by M. Alcubierre (1994). These warp drives are constructs that allow some object (a spacecraft) to travel at superluminal velocities by manipulating space-time in a way such that the spacecraft never locally exceeds the speed of light. However in a manner identical to the inflationary stage of the universe, the spacecraft does have a relative speed, defined as change of proper spatial distance over proper spatial time, faster than the speed of light.

Interest in warp drives has not been solely confined to the realm of theoretical speculation as shown by the formation of the NASA Breakthrough Propulsion Program and the British Aerospace Project Greenglow, both of whose purpose has been to investigate the realization of these ideas.

In the spirit of Morris, Thorne and Yurtsever (1988) these warp drives, although highly speculative in nature, provide an unique and inspiring opportunity to ask the question `what constraints do the laws of physics place on the abilities of an arbitrarily advanced civilization'. In this paper a new and innovative mechanism to generate the necessary `Alcubierre warp bubble' is proposed. The main focus of the paper is to demonstrate that the manipulation of the radius of one (or more) of the extra dimensions found in quantum gravity theories, creates a local asymmetry in the cosmological constant which could be used to propel a space vehicle.

At such an early stage in the theoretical development of this idea it is challenging to make predictions on how this 'warp drive' might function. Naively one could envision a spacecraft with an exotic power generator that could create the necessary energies to locally manipulate the extra dimension(s). In this way, an advanced spacecraft would expand/contract the compactified spacetime around it, thereby creating the propulsion effect.

To appreciate this new propulsion mechanism there are several areas of physics that must first be reviewed. These will be discussed in the following section.

### The Cosmological Constant

Current observations of distant supernova (Garnavich, 1998) indicate that the universe is expanding at an *increasing* rate. This expansion is realized in Einstein's equations as the cosmological constant term. It is, in essence, a 'fix' in

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Einstein's Field equations that help match the theory with observations. This 'Lambda' term, as it is known, provides a necessary addition to the gravitational field to correlate with what we see in the cosmos.

The fundamental nature of this Lambda term is still a mystery, nearly a century after its introduction into cosmology. Physicists are not certain *what* creates the additional component to the gravitational field; we simply know that it is there. Several ideas exist as to the nature of this field, dark energy, for example, is a popular contemporary phrase for the lambda term. Efforts have been made to explain the cosmological constant using the more modern quantum field theory, created after Einstein and his gravitational equations.

## Quantum Field Theory

Quantum field theory (QFT) is widely regarded as the most successful physical theory of all time. Its predictions are precise to many decimal places and no experiment has ever contradicted the predictions of the theory. With this said, it would seem only natural to try to account for the cosmological constant using QFT. The calculations (Cherednikov, 2002) are based on summing all the zero point oscillations with a Planck scale cut-off which would give us an estimate to the overall vacuum energy density, which is equivalent to calculating the cosmological constant.

$$\langle E_{vac} \rangle = \frac{1}{2}\int_0^M \frac{d^3k}{(2\pi)^3}\sqrt{k^2+m^2} \approx \frac{M^4}{16\pi^2}. \tag{1}$$

The energy density predicted using this equation is of order $10^{71}$ GeV$^4$, which conflicts with the observed value of $10^{-48}$ GeV$^4$. This is by far the worst predication of theoretical physics and is off by a factor of $10^{119}$. This failure of quantum theory has recently been reexamined using brane world scenarios born of string theory (Ellwanger, 2005; Binetruy and Deffayet and Ellwanger and Langlois 2000; Kanti and Kogan and Olive and Pospelov, 2000; Alwis, 2001).

## Brane World Models

QFT and Einstein's General Relativity (GR) have long been known to be incompatible. When one attempts to describe gravity using QFT one generally obtains meaningless results. Similarly one cannot often use the equations of GR to perform quantum calculations.

String theory is a modern and valiant attempt to create a quantum theory of gravity, in essence, to unify GR and QFT under one simple framework. The fundamental 'object' from which all matter and energy arises is a vibrating string, a strand of energy, which can acquire varying vibrational modes and patterns which manifest themselves as the bosons and fermions we are familiar with.

One highly popularized by product of this unification is the prediction that we do not inhabit a universe of three spatial and one temporal dimensions, but that there are, in fact, extra dimensions, too small to probe with today's technology.

The M-theory extension to strings utilizes the idea of "branes" in addition to strings. In brane world models, our large universe is a '3 brane' which exists in a higher dimensional 'bulk'.

Utilizing brane world scenarios it is possible to *decrease* the vacuum energy which was found to give a prediction of the cosmological constant far in excess of that observed, even in the case of assumed supersymmetry breaking at the TeV scale.

## Supersymmetry

Supersymmetry (SUSY) is a theoretical development of particle physics which postulates that every boson and fermion has a 'partner' particle, called its 'superpartner'. The theory developed in the 1970s and was motivated mainly by a need to suppress the quadratic divergence (infinity) associated with calculating the mass squared of the

Higgs boson. To date, no superpartner has been discovered. This implies that supersymmetry is somehow 'broken' in our universe. Consider, for example, beta decay where a single electron is ejected from a nucleus. If supersymmetry were 'preserved' there would be an equal chance of us seeing the supersymmetric partner of the electron, the 'selectron' being ejected from the nucleus. As this is clearly not the case, it is presumed that supersymmetric partners are far more massive than the standard particles that we are familiar with, and require more energy to create. This is why we say the supersymmetry is broken.

## The Chen and Gu approach

To summarize what has been discussed so far, there exists a cosmological constant whose value cannot be predicted with any accuracy using quantum field theory. The Chen and Gu approach 'tames' the QFT calculation by an almost exact cancellation of fields in *our* universe by fields in the *bulk* (Chen and Gu, 2004). The basic approach is that supersymmetric contributions to the vacuum energy actually *subtract* from the contributions in our 3 brane, thereby reducing the overall vacuum energy calculations to a value that more closely agrees with observation. Review the fundamental elements, the Chen and Gu approach supposes that:

- we exists in a 3 brane in which supersymmetry is *broken*
- our 3 brane exists in a higher dimensional bulk in which supersymmetry is *not broken*
- our 3 brane has an inherent 'thickness' that 'bulges' into the bulk
- supersymmetric fields in the bulk almost perfectly cancel fields in the 3 brane.

The new approach to 'warp drive', which this paper exploits, is that the vacuum energy is, in fact a function of the size (radius) of the extra dimensions. Recall that the vacuum energy is what is also referred to as the cosmological constant in General Relativity. This vacuum energy directly effects space and is responsible for its current expansion. The main theme of this paper is that if one could locally manipulate the size of an extra compactified dimension then, in theory, one could expand/contract space-time at will.

## Technical Setup

In the context of brane-world models our universe is a (3+1) brane residing in some higher dimensional bulk (Brax, Van de Bruck and Davis, 2004). It is known phenomenologically that supersymmetry is broken on our 3-brane, however is has been suggested that it may not be broken on the bulk (Giudice and Rattazzi, 1999).

Unbroken SUSY decrees that the components of the Chiral or Gauge Multiplets respectively share equal masses in the bulk and have the same interaction strength. However on the 3-brane SUSY breaking induces a mass square difference between them. Motivated by string theory the 3-brane has an effective thickness characterized by the string length $l_s$. As a result the Casimir energy is non-trivial in the extra dimensional volume that encompasses the brane. This energy has the necessary features to account for the cosmological constant (Aghababaie and Burgess and Parameswaran and Quevedo, (2004).

For simplicity assume an $M^4$ manifold with a toroidal compact extra dimension with radius a. SUSY breaking around the brane alters the Casimir energy which leads to a mass shift of the bulk fields. It is the aim of this paper to demonstrate that the mass shift is directly related to the radius of the extra dimension and, as such, a local change in the radius of the extra dimension will have the effect of altering the mass shift and thus the Casimir Energy which locally effects the value of the cosmological constant in the region effectively creating a 'bubble' of inflation/contraction.

A spacecraft with the ability to create such a bubble will always move inside its own local light-cone. However the ship can utilize the expansion of space-time behind the ship to move away from some object at any desired speed or equivalently to contract the space-time in front of the ship to approach any object.

## Theory

To build the model we consider for simplicity a scalar field and its SUSY partner a `calar' field. The scalar part of the action will be:

$$S = \int d^4x d^n y \sqrt{|g|} \left[ \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} (m_0^2 + \Delta m^2 \phi^2) \right], \tag{2}$$

where:

$$\Delta m^2 (y) = m^2 e^{-\frac{2|y|}{\delta}}, \tag{3}$$

characterizes the mass-square shift and the location of the three-brane is at y=0. The shift of the Casimir energy density due to SUSY-breaking in the three brane 'extra' volume is (Chen and Gu, 2004):

$$\delta\rho_\nu (\Delta m^2, a) = \rho_\nu^{(ren)} (m^2, a) - \rho_\nu^{(ren)} (m^2 = 0, a)$$

$$\cong \kappa_n \frac{a^2}{a^{4+n}} \Delta m^2 (y), \tag{4}$$

where $\kappa_n$ is some constant. The renormalized Casimir energy density is:

$$\rho_\nu^{(ren)} (m^2, a) = \rho_\nu (m^2, a) - \rho_\nu (m^2, a \to \infty). \tag{5}$$

After integrating over the extra dimensional space the renormalized Casimir energy density in 4-dimensional space-time is:

$$\delta\rho_\nu^4 \cong \kappa_n \frac{m^2}{a^2} \frac{\pi^{\frac{n}{2}} \Gamma(n)}{2^{n-1} \Gamma^{\frac{n}{2}}} \left( \frac{\delta}{a} \right)^n, \tag{6}$$

where $\delta$ is the width of the brane and 'a' is the radius of the compactified extra dimension.

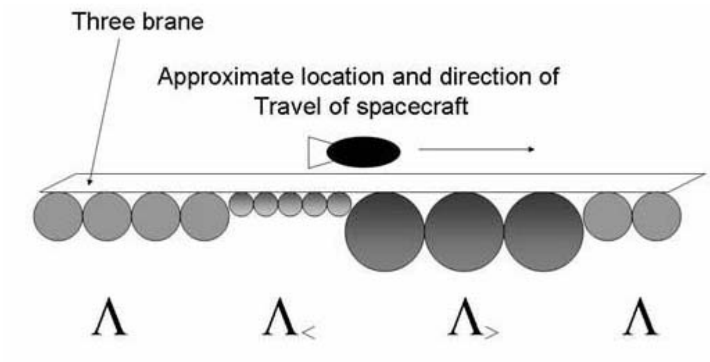
The same calculations can be applied to the super-partner calar field. The only change is a sign difference. Thus the total Casimir energy density contribution from a scalar/calar field induced by SUSY breaking is:

$$\delta\rho_{total} = \delta\rho_{scalar} (\Delta m^2, a) + \delta\rho_{calar} (\Delta \tilde{m}^2, a), \tag{7}$$

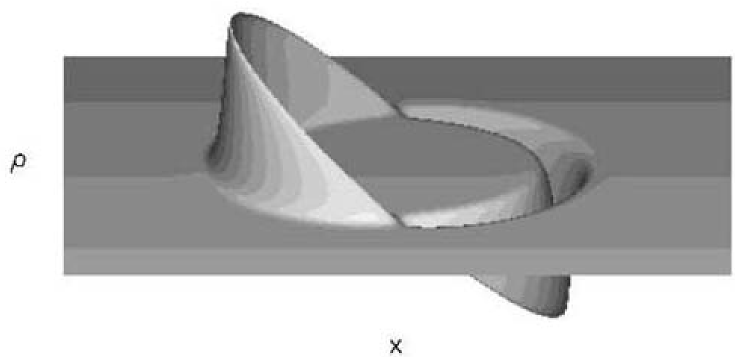
$$= \delta\rho_{scalar} (\Delta m^2, a) - \delta\rho_{scalar} (\Delta \tilde{m}^2, a). \tag{8}$$

It is important to note that although the Casimir energy lies in the extra dimension it does contribute to the overall energy density of the universe in $M^4$ and represents the $\Lambda$ term in Einstein's equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{9}$$




**(a)** Circles Represent the Radius of the Extra Dimension. The Spacecraft Locally *increases/decreases* the Radius at Its Front/Rear. The Resulting Shift in the Cosmological Constant Drives the Spacecraft Forward.




**(a)** The Aclubierre Top-Hat Metric Is Shown for Comparison.

**FIGURE 1.** Top Image Illustrates the Propulsion of a Spacecraft through Local Alteration of the Radius of the Extra Dimension. Below Is the Familiar Alcubierre Metric.

The important term in equation (5) is $(\delta\backslash a)^n$ which demonstrates that the Casimir energy density and hence the cosmological constant term, are immutably related to the radius 'a' of the extra dimension.

Consider some arbitrarily advanced civilization with the ability to locally alter the radius of the extra dimension as a function of their position. This would bring about a local shift in the cosmological constant roughly of the form:

$$\Delta\Lambda \cong \kappa_n \frac{m^2}{a^2} \frac{\pi^{\frac{n}{2}} \Gamma(n) \delta^n}{2^{n-1} \Gamma^{\frac{n}{2}}} \left[ \left(\frac{1}{a}\right)^n - \left(\frac{1}{a'}\right)^n \right]. \tag{10}$$

Here, a' represents the modified radius. Figure 1 illustrates the basic idea. The radius of an extra dimension is shown as circles of smaller/larger radius in the immediate vicinity of a spacecraft when compared against the unchanged extra dimension just beyond the spacecraft. The effect is to *locally* alter the value of Λ, the cosmological constant. The Alcubierre top-hat metric is shown beneath to demonstrate the analogous effects on space-time.

## CONCLUSIONS

This paper has introduced a novel method of spacecraft propulsion that could potentially revolutionize space exploration. The idea is, however extremely theoretical in that we have not yet discovered:

- Supersymmetric particles
- Extra dimensions
- Branes

These ideas are certainly at the forefront of theoretical physics and evidence for supersymmetry and extra dimensions may be revealed in the next generation of particle accelerator to open at CERN in 2007.

In the meantime, there are certainly avenues of theoretical research, based on this theory, that can be followed. First and foremost is the idea that a compact dimension can be manipulated locally, i.e. its radius can be modified by some technological means. String theory suggests that dimensions are held compact by strings wrapping around them (Cleaver and Rosenthal, 1995; Brandenberger and Vafa, 1989; Tseytlin and Vafa, 1992). If this is, indeed the case, then it may be possible to increase or decrease the string tension, or even annihilate string winding modes. This would achieve the desired effect of changing the size of the extra dimensions which would lead to propulsion under this model. It would thus be prudent to research this area further and perform calculations as to the energies required to affect an extra dimension and to try and relate this energy to the acceleration a spacecraft might experience. It is the ambition of the author to expand on these ideas in future papers.

## NOMENCLATURE

$a$ = radius of extra dimension (m)
$E_{vac}$ = vacuum energy density ($GeV^4$)
$g$ = measure
$G$ = gravitational constant ($N.m^2.Kg^{-2}$)
$k$ = wave vector ($m^{-1}$)
$m$ = mass of field (GeV)
$S$ = Action (GeV)
$R$ = Ricci scalar
$y$ = location of 3 brane within bulk (m)
$M^4$ = Minkowski space
$T^n$ = toroidal space
$R_{\mu\nu}$ = Ricci tensor ($GeV^4$)
$T_{\mu\nu}$ = stress energy tensor ($GeV^4$)
$\varphi$ = field
$\delta$ = brane thickness (m)
$\rho_\upsilon$ = vacuum energy density ($GeV^4$)
$\kappa_n$ = constant
$\Gamma$ = Gamma function
$\delta\rho$ = shift in vacuum energy density ($GeV^4$)
$g_{\mu\nu}$ = metric tensor (m)
$\Lambda$ = cosmological constant ($m^{-2}$)

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